Midterm Introduction to Logic (AI and MA + Guests)

Wednesday 12 December, 2018, 9 – 11 AM

Instructions: Read Carefully

☞ Put the version of the course that you are registered in at the top of the first page (either "AI" or "Math + Guests").

☞ Only write your student number at the top of the exam, not your name, so that we can grade anonymously. Also put your student number at the top of any additional pages.

☞ Put the name of your tutorial group (AI 1, AI 2, …or MG 1, MG 2, MG 3) at the top of the exam and at the top of any additional pages.

☞ Leave the first ten lines of the first page blank (for the calculation of your grade).

☞ Use a blue or black pen (so no pencils, no red pens).

☞ With the regular exercises, you can earn 90 points. By writing your student number and tutorial group on all pages, you earn a first 'free' 10 points. With the bonus exercise, you can earn an additional 10 points. The total grade is: *(the number of points you earned with the regular and bonus exercises + the first 'free' 10) divided by 10, with a maximum grade of 10.*

Good Luck!

1: Translation into propositional logic (10 points) Translate the following sentences into *propositional logic*. Atomic sentences are represented by uppercase letters. Do not forget to provide the translation key — one key for the whole exercise. Represent as much logical structure as possible.

a. The child likes Lego only if she does not like Playmobil.

b. If the child likes wooden toys, then she likes neither Lego nor Playmobil.

2: Translation into first-order logic (10 points) Translate the following sentences to *first-order logic*. Do not forget to provide the translation key — one key for the whole exercise. Represent as much logical structure as possible.

a. The Linnaeusborg is taller than the Bernoulliborg but the Bernoulliborg is easier to navigate than both the Linneausborg and Nijenborgh.

b. The Linneausborg is easier to navigate than Nijenborgh if and only if neither Nijenborgh nor the Bernoulliborg is red.

3: Formal proofs (30 points) Give formal proofs of the following inferences. Do not forget to provide justifications in the correct order. You may only use the Introduction and Elimination rules and the Reiteration rule.

a.

| $a = b \wedge c = b$
| $\neg P(a)$
|—
| $\neg P(c)$

b.

|—
| $(\neg Q \to Q) \to \neg\neg Q$

c.

| $A \vee \neg C$
| $C \vee \neg B$
|—
| $B \to A$

4: Truth tables (15 points) Use *truth tables* to answer the next questions. Make the full truth tables, and do not forget to draw explicit conclusions from the truth tables in order to explain your answers. Order the rows in the truth tables as follows:

P	Q	R	...
T	T	T	...
T	T	F	...
T	F	T	...
T	F	F	...
F	T	T	...
F	T	F	...
F	F	T	...
F	F	F	...

a = b	b = c	c = a	...
T	T	T	...
T	T	F	...
T	F	T	...
T	F	F	...
F	T	T	...
F	T	F	...
F	F	T	...
F	F	F	...

a. Is $((P \leftrightarrow Q) \leftrightarrow \neg R) \leftrightarrow ((P \leftrightarrow Q) \wedge (Q \leftrightarrow \neg R))$ a *tautology*?

b. Is the sentence $(a = b \wedge \neg(b = c)) \rightarrow \neg(c = a)$ a *logical truth*?
Explain your answer and indicate the spurious rows in the truth table.

5: Normal forms of propositional logic (15 points)

a. Provide a negation normal form (NNF) of this sentence:
$(\neg Q \vee R) \rightarrow \neg(\neg P \leftrightarrow Q)$

b. Provide a disjunctive normal form (DNF) of this sentence:
$\neg((Q \rightarrow R) \wedge (R \rightarrow P)) \vee \neg(P \rightarrow Q)$

Indicate the intermediate steps. You do not have to provide justifications for the steps.

6: Set theory (10 points) Consider the following three sets:

$$A = \{\emptyset, a\}, \quad B = \{\emptyset, \{a\}\} \text{ and } C = \{\emptyset, \{a\}, \{\emptyset, \{a\}\}\}.$$

For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

a. $a \in A$

b. $A \subseteq B$

c. $A \cap B = \emptyset$

d. $B \cap C = B$

e. $A \subseteq C$

f. $C = B \cup \{B\}$

g. $\emptyset \in A \cap B \cap C$

h. $(B \backslash A) = \{a, \{a\}\}$

i. $C \subseteq A \cup B$

j. $B \in C \backslash A$

7: Bonus question (10 points) Give a formal proof of the following inference:

$P \leftrightarrow Q$
$\neg(Q \leftrightarrow R)$

$P \leftrightarrow \neg R$

Do not forget to provide justifications in the correct order. You may only use the Introduction and Elimination rules and the Reiteration rule.